# Phenomenological Treatment of the $\rho$ Trajectory*

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A phenomenological method is applied to the $\rho$ trajectory. It is argued that once the intercept is known, the method is expected to give reliable information about the trajectory in the region of interest for high-energy scattering. Solutions for different values of the intercept ranging from 0.3 to 0.8 are given. From certain general requirements the higher intercepts seem to be favored.

IN a previous paper, we presented a phenomenological method for calculating Regge trajectories.[1] The approach was based on the real-analyticity and threshold behavior as well as on the available experimental information, and the first application was to the Pomeranchuk trajectory. It is our purpose in this paper to obtain by the same approach the approximate form for the $\rho$ trajectory.

Our starting point is a four-parameter ansatz for the imaginary part of the trajectory function $\alpha(t)$, namely,

$$\mathrm{Im}\alpha(t)=C\nu^{\lambda}/[C_1+(C_2-\nu)^2] \quad \text{for} \quad \nu>0,$$

and

$$\mathrm{Im}\alpha(t)=0 \quad \text{for} \quad \nu<0, \tag{1}$$

where $\nu=\frac{1}{4}(t-t_0)$, and $t_0=4m_\pi^2=4$. Using the usual dispersion relation for $\alpha$, we have

$$\mathrm{Re}\alpha(\nu)=\alpha(\nu=-1)+\frac{C(\nu+1)}{\pi}\times P\int_0^\infty \frac{\nu'^{\lambda}d\nu'}{(\nu'-\nu)[C_1+(C_2-\nu)^2](\nu'+1)}, \tag{2}$$

where for convenience we have made the subtraction at $\nu=-1$, the point corresponding to the forward direction in the crossed channel. This integral can be evaluated by applying Cauchy's theorem to Eq. (1). The result is

$$\alpha(\nu)=\alpha(\nu=-1)-\frac{(\nu+1)C}{\sin\pi\lambda}\times\left\{\frac{\nu^{\lambda}e^{-i\pi\lambda}}{(\nu+1)[C_1+(C_2-\nu)^2]}-\frac{a^{\lambda}e^{-i\pi\lambda}}{(a-b)(a+1)(\nu-a)}-\frac{b^{\lambda}e^{-i\pi\lambda}}{(b-a)(b+1)(\nu-b)}-\frac{1}{(\nu+1)[C_1+(C_2+1)^2]}\right\}, \tag{3}$$

where

$$a=-C_2+i(C_1)^{1/2}, \quad \text{and} \quad b=-C_2-i(C_1)^{1/2}.$$

In Eq. (3), the four parameters $\lambda$, $C$, $C_1$, and $C_2$ are determined with the help of the four conditions

(i) $$\alpha(\infty)=-1,$$

(ii) $$\Gamma_\rho=\left(\frac{\mathrm{Im}\alpha(t)}{[d\,\mathrm{Re}\alpha(t)/dt](t)^{1/2}}\right)_{t=m_\rho^2}\approx 100\text{ MeV},$$ (see Ref. 2),

(iii) $$\lambda=\alpha(\nu=0)+\tfrac{1}{2},$$ (see Ref. 3),

and

(iv) $$\mathrm{Re}\alpha(t=m_\rho^2)=1.$$

These conditions, when imposed on Eq. (3), and when $\alpha(\nu=-1)$ is known, determine the four parameters. The solution was found with the help of the IBM 7094 computer of the Lawrence Radiation Laboratory. Since the intercept value $\alpha(\nu=-1)$ is not well known, we take a range of values from 0.3 to 0.8 and attempt to choose the most plausible solution among them. We should also mention that condition (i) is questionable. All that is known is that $\alpha(\infty)<1$. However, taking $\alpha(\infty)=0$, for example, makes not too significant a change in the general features of trajectory in the region $-10m_\pi^2<\nu<10m_\pi^2$. Figure 1 shows $\mathrm{Re}\alpha$ versus $\nu$ for intercepts of 0.3 to 0.8, and for the sake of comparison, $\mathrm{Re}\alpha$ for the Pomeranchuk trajectory of Ref. 1 is also given.

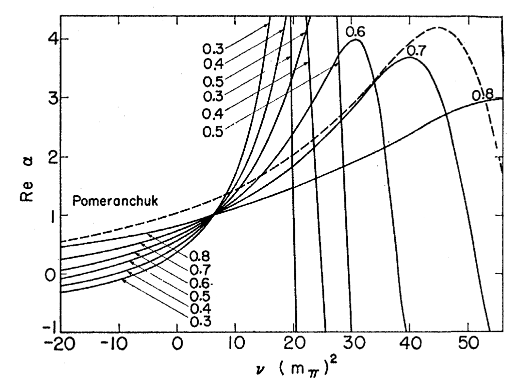


FIG. 1. $\mathrm{Re}\alpha$ versus $\nu$ for intercepts of 0.3 to 0.8. The dashed curve is $\mathrm{Re}\alpha$ versus $\nu$ for the Pomeranchuk trajectory given in Ref. 1.

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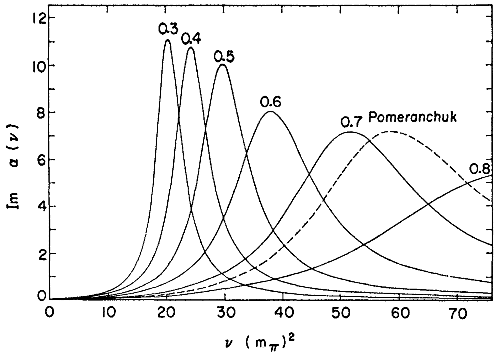


FIG. 2. Im$\alpha$ versus $\nu$ corresponding to the curves in Fig. 1. The intercept values are indicated on these curves.

Figure 2 is a plot of the corresponding Im$\alpha$. It is seen from these curves that for smaller intercepts the imaginary part of $\alpha$ has a sharper maximum and Re$\alpha$ reaches a higher maximum value. In particular, for the small intercepts, the solution gives rise to a spin-3 recurrence of a width narrow enough to be experimentally observable. These features are examined in Figs. 3, 3(a) showing the relation between the intercept and the width $\Gamma_R$ of the spin-3 recurrence, and 3(b) showing the relation between the mass and the width of the spin-3 recurrence. It is clear from Fig. 3(b) that if such a recurrence exists at all, its mass should be smaller than 2 BeV so an experimental search may be correspondingly restricted. Once the mass is experimentally known, the width is predicted by this curve. Figure 3(c) gives the slope of the trajectory at $t=0$ versus the intercept value. The position and the width of the spin-3 recurrence are given as running parameters. If the fact that no recurrence has been found is interpreted to mean that the width is too large to have been observed, then the higher intercepts of 0.7 to 0.8 are favored. We should mention that the solution with intercept of 0.7 and slope $(d\alpha/dt)_{t=0}\approx 0.44$ BeV$^{-2}$ is in fair agreement with the results of Scotti and Wong[4] concerning the low-energy nucleon-nucleon scattering and with the results of Brandsen *et al.*[5] concerning the strip approximation in $\pi-\pi$ scattering. Also this solution gives essentially the same slope as the Pomeranchuk trajectory (see Fig. 1). Finally, Table I gives the values of the parameters $C$, $C_1$, $C_2$, and $\lambda$.

TABLE I. Values of $C_1$, $C_2$, $\lambda$, and $C$.

| $\alpha(\nu=-1)$ | $C_1$ | $C_2$ | $\lambda$ | $C$ |
|---|---|---|---|---|
| 0.3 | 6.798 | 20.26 | 0.8750 | 5.443 |
| 0.4 | 11.99 | 24.04 | 0.9670 | 5.928 |
| 0.5 | 23.38 | 29.29 | 1.058 | 6.550 |
| 0.6 | 67.98 | 37.06 | 1.148 | 8.478 |
| 0.7 | 173.8 | 49.53 | 1.2373 | 9.669 |
| 0.8 | 766.9 | 72.73 | 1.3256 | 13.31 |
| Pomeranchuk | | | | |
| 1.0 | 259.7 | 55.24 | 1.533 | 3.79 |

In conclusion, we should like to make a few remarks about the sensitivity of our results to the conditions (i) and (ii). If we replace condition (i) by $\alpha(\infty)=-2$, the solution for Re$\alpha$ in the region $-10m_\pi^2<\nu<10m_\pi^2$ changes very little; however, $\Gamma_R$, $M_R$ and Re$\alpha_{\max}$ change considerably ($\Gamma_R$ gets smaller, $M_R$ and Re$\alpha_{\max}$ get bigger). In condition (ii), $\Gamma_\rho$ is not experimentally known very accurately. If we choose $\Gamma_\rho=80$ MeV, say, then again $\Gamma_R$, $M_R$, and Re$\alpha_{\max}$ change considerably ($\Gamma_R$ gets smaller, $M_R$ and Re$\alpha_{\max}$ get bigger). Here again, Re$\alpha$ and its derivative change very little in the region $-10m_\pi^2<\nu<10m_\pi^2$. Consequently, once the intercept is known, our treatment is expected to give reliable information about the trajectory in the region $-10_\pi^2<\nu<10m_\pi^2$. This is the region of interest in high-energy scattering.

From the present calculation based on conditions (i) and (ii), we have seen that higher intercepts are favored. On the other hand, as pointed out by Phillips,[6] if the $\rho$ exchange is to play a dominant role in $np$ charge exchange scattering then the experiment of Palevsky *et al.*,[7] who measured the energy dependence at $t=0$,

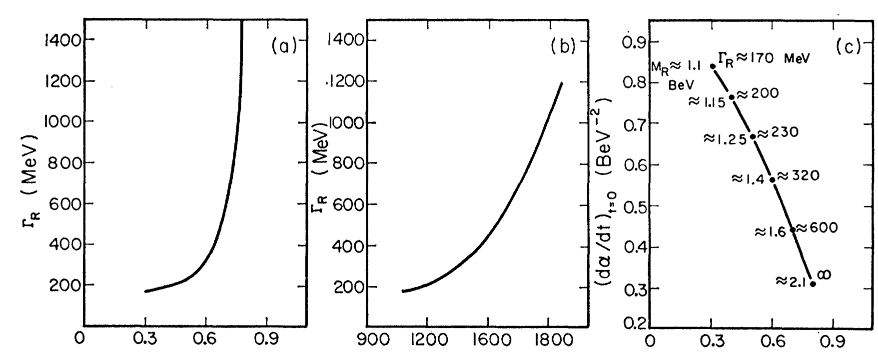


FIG. 3. (a) $\Gamma_R$ versus $\alpha(\nu=-1)$. (b) $\Gamma_R$ versus $M_R$. (c) $d\alpha/dt(t=0)$ versus $\alpha(t=0)$.

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would require an intercept of about 0.3. This value is in sharp conflict with the higher intercepts of 0.7 and 0.8, which we have favored here.

Recently, Abolins *et al.*[8] have reported evidence for a resonance at 1.22 BeV that may have the same spin and quantum numbers as the $\rho$. If so, there would be a second $\rho$ trajectory with a smaller intercept at $t=0$ than the first. The combined result might be an "average" intercept of about 0.3 as required by the experiment of Palevsky *et al.*[7] (If the resonance at 1.22 BeV has spin 3 and is the recurrence of the $\rho$, it would roughly fit into our solution with the intercept of 0.5.)

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# Interrelations Among Dissymmetries in $SU_3$ Supermultiplets

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The mass splittings within the (unitary) supermultiplets of the strongly-interacting particles are studied in the octuplet model of $SU_3$ symmetry. Using arguments based on the requirement of dynamical self-consistency, qualitative properties of the first-order perturbations in masses and coupling constants are studied in the ladder approximation. Further evidence for the origin of the particular type of dissymmetry leading to the Gell-Mann–Okubo mass formula and to isotopic spin symmetry is gotten through the relations between the dynamical effects in different supermultiplets. The observed mass splittings in the pseudoscalar meson, vector meson, and the $\frac{1}{2}(+)$ baryon octuplets, as well as in the $\frac{3}{2}(+)$ baryon decuplet, are compatible with the general features expected from first-order perturbations. Some higher order perturbations in the $\frac{3}{2}(+)$ decuplet are also discussed.

## I. INTRODUCTION

THE concept of the strongly interacting particles as self-consistent bound states has been applied in an earlier paper[1] to a study of the deviations from $SU_3$ symmetry in a model containing only vector mesons. We shall discuss here, on the same basis, a more realistic model in which several kinds of particles enter. We shall make use of the following points which were discussed in detail in our previous work. We begin with a self-consistent set of particles which incorporates full symmetry, so the search for additional sets of self-consistent particle masses can be carried out through calculations which make use of the techniques of ordinary perturbation theory. If a first-order perturbation having a particular transformation character approximately reproduces itself, there will be another self-consistent set of particles with a small dissymmetry of the given type; the magnitude of this dissymmetry is then fixed by the self-consistency requirement, but depends on the higher order terms. Moreover, if this self-generating dissymmetry corresponds to a (1,1) (or 8-fold) tensor, general characteristics of the higher order terms imply the necessary maintenance of isotopic spin symmetry. In the vector-meson model, our estimates of the effects of perturbations did indeed favor the (8) dissymmetry, and we shall show that the same result is obtained here as well.

It is sufficient for us to consider here only perturbations which retain isotopic spin invariance, because in first order only the $SU_3$-multiplet character of the perturbation is relevant, and because of the fact that a self-consistent (8) perturbation necessarily retains $SU_2$ invariance. The normalized charge-independent mass deviations in the baryon octuplet are listed in Tables I and II for the possible dissymmetries.

Our attention in this paper will be given mainly to the baryon states, but we first remark on the inter-

TABLE I. Mass deviations in octuplets.

| $Y, T$ | $1, \frac{1}{2}$ | $0, 1$ | $0, 0$ | $-1, \frac{1}{2}$ | Normalization |
|---|---|---|---|---|---|
| (8)+ | 1 | −2 | 2 | 1 | $2\ (5)^{1/2}$ |
| (8)− | −1 | 0 | 0 | 1 | 2 |
| (27) | 3 | −1 | −9 | 3 | $2\ (30)^{1/2}$ |

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[1] R. E. Cutkosky and Pekka Tarjanne, Phys. Rev. **132**, 1888 (1963), hereafter cited as I. The present paper contains additional references.